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A TWO DIMENSION NUMERICAL MODEL FOR WAVES INDUCED CURRENTS IN THE COASTAL ZONE

LE XUAN HOAN AND NGUYEN MANH HUNG
Institute of Mechanics, NCST of Vietnam

ABSTRACT. In this paper, the results of two dimension numerical model for waves induced currents in the coastal zone are presented. The model was adjusted and verified at Phan Ri-Binh Thuan zone. The results of the testing show a good agreement between the observed and computed values. Then the model is applied to calculation of the distribution of waves induced currents in Ly Hoa area.

1. Introduction

Waves and wave induced currents are a leading cause of the processes of sediment transport and shoreline changes. It is necessary to know the distribution of waves induced currents for calculating and forecasting the bottom and shoreline changes. However, up to now, in Vietnam, the calculation of waves induced currents has been considered not much. The Longuet-Higgins's one dimension model is usually used to calculate the waves induced currents in research subjects.

The radiation stress is the leading external force creating waves induced currents. Therefore it is necessary to know the distribution of wave parameters in the shallow water to calculate the radiation stress. The RCPWAVE model built at the Centre for Marine Environment Survey, Research and Consultation (CMESRC), Hanoi Vietnam is used to calculate the distribution of wave parameters in the shallow water [3]. The reflection is not considered in the RCPWAVE model therefore the two dimension numerical model for waves induced currents is only used for light slope bottom zones

2. Basic equation, initial and boundary conditions [1]

+ **Wave Diffraction behind a coastal structure**

Wave diffraction behind a coastal structure is obtained by the equation:

$$K_{diff} = \frac{H_{tip}}{H_d}$$

where:

- K_{diff} diffraction coefficient,
- H_{tip} wave height in the tip of the structure,

- H_d wave height in the point to be calculated.

Diffraction coefficient in turn is the function of the angle between wave direction and the structure, the angle between wave direction and the point of calculation based on the tip of the structure and the ratio between the distance from the point of calculation to the tip of structure to wave length at the tip of structure. This diffraction coefficients for each point of calculation behind the structure is derived by the Sommerfeld solution of optical diffraction applied by Penny and Price [3].

+ Wave induced current

The equation for conservation of momentum:

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=-g\frac{\partial \zeta}{\partial x}-\frac{\tau_{bx}}{\rho h}-\frac{1}{\rho h}\left(\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y}\right)$$
$$\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}=-g\frac{\partial \zeta}{\partial y}-\frac{\tau_{by}}{\rho h}-\frac{1}{\rho h}\left(\frac{\partial S_{yy}}{\partial y}+\frac{\partial S_{xy}}{\partial x}\right)$$

The equation for conservation of mass:

$$\frac{\partial \zeta}{\partial t}+\frac{\partial U_h}{\partial x}+\frac{\partial V_h}{\partial y}=0$$

where:

- U, V - depth-averaged components of velocity (m/s) in the x- direction and the y- direction, they are defined as follows:

$$U=\frac{1}{h}\int_{-h}^{\zeta}u(z)dz,\quad V=\frac{1}{h}\int_{-h}^{\zeta}v(z)dz$$

- $\zeta(x,y,t)$ - mean sea level in existential waves compared with still level (m),
- d - water depth above reference bottom (m),
- h - total water depth (m), ($h=d+\zeta$),
- g - gravity acceleration [m/s^2],
- ρ - density of water (kg/m^3),
- t - time,
- τ_{bx}, τ_{by} - components of bottom friction stress in x-direction and y-direction (kg/ms^2), they are expressed by quadratic forms:

$$\tau_{bx}=\rho C_f U\sqrt{U^2+V^2};$$
$$\tau_{by}=\rho C_f V\sqrt{U^2+V^2}$$

where C_f is the bottom friction coefficient in a coexistent wave-current field.

- S_{xx}, S_{yy}, S_{xy} - components of radiation stress (kg/ms^2), they are defined as follows:

$$S_{xx} = E\Big(\frac{3}{2}n - \frac{1}{2}\Big) + E\frac{n}{2}\cos 2\alpha\,;$$
$$S_{yy} = E\Big(\frac{3}{2}n - \frac{1}{2}\Big) - E\frac{n}{2}\cos 2\alpha\,;$$
$$S_{xy} = En\sin\alpha\cos\alpha\,;$$
$$n = \frac{C_g}{C} = 0.5\Big[1 + \frac{4\pi h/L}{\sinh(4\pi h/L)}\Big]\,;$$
$$E = \frac{\rho g H^2}{8}\,;$$
$$L = \frac{gT^2}{2\pi}\tanh\frac{2\pi h}{L}\,,$$

where: E - the wave energy, H - the significant wave height, C_g - the group velocity, C - the phase velocity, L - the wave length, α - the angle between the approach wave direction and the normal of shoreline.

The Coriolis component is neglected because the model is applied only for narrow zones.

Initial conditions: $U = 0$, $V = 0$, $\zeta = 0$

Boundary conditions:

- At the solid boundary: $U = 0$, $V = 0$
- At the lateral boundary: $\dfrac{\partial U}{\partial x} = 0$, $\dfrac{\partial \zeta}{\partial x} = 0$ (no longshore gradients of the longshore current and average water levels)
- At the open-sea boundary: $U = V = \zeta = 0$ (The open-sea boundary is located in a sufficiently deep region, where zero nearshore current velocities and no changing of mean water level can be assumed)

The stable condition:

$$\frac{C\Delta t}{\Delta x} < 1, \quad (C = \sqrt{gh})$$

where C is the phase velocity in the shallow water.

The steady state condition:

In each time-step, the total kinetic energy in the flow domain is computed from the sum:

$$E^n = \sum_i\sum_j \Big\{\big[\big(U^n_{i,j} + U^n_{i+1,j}\big)^2 + \big(V^n_{i,j} + V^n_{i,j+1}\big)^2\big]\frac{h_{i,j}\Delta x\Delta y}{8}\Big\}.$$

The steady state is reached when the ratio $\dfrac{|E^{n+1} - E^n|}{E^{n+1}}$ become less than a test convergence.

3. Algorism and difference scheme

For simple calculating procedure, FTCS method is used, which is a finite difference based on scheme with fore time and centre spatial step.

The parameter θ in the recommended diffusive Lax type finite difference for the time derivation is used:

$$\frac{\partial U}{\partial t}=\frac{\left[U_{i,j}^{n+1}-\theta U_{i,j}^{n}+(1-\theta)(U_{i+1,j}^{n}+U_{i-1,j}^{n}+U_{i,j+1}^{n}+U_{i,j-1}^{n})/4\right]}{\Delta t}$$

The equations (1.1), (1.2), (1.3) are differenced as follows:

$$\begin{aligned}U_{i,j}^{n+1}&=\theta U_{i,j}^{n}+(1-\theta)\frac{1}{4}(U_{i+1,j}^{n}+U_{i-1,j}^{n}+U_{i,j+1}^{n}+U_{i,j-1}^{n})-\frac{\Delta t}{2\Delta x}U_{i,j}^{n}(U_{i+1,j}^{n}-U_{i-1,j}^{n})\\&-\frac{\Delta t}{2\Delta y}\overline{V}_{ij}^{n}(U_{i,j+1}^{n}-U_{i,j-1}^{n})-g\frac{\Delta t}{2\Delta x}(\zeta_{i+1,j}^{n}-\zeta_{i-1,j}^{n})-\frac{\tau_{hx}^{n}}{\rho h}\\&-\frac{1}{\rho h}\Big[\frac{\Delta t}{2\Delta x}(S_{xx_{i+1,j}}-S_{xx_{i-1,j}})+\frac{\Delta t}{2\Delta y}(S_{xy_{i,j+1}}-S_{xy_{i,j-1}})\Big]\end{aligned}$$

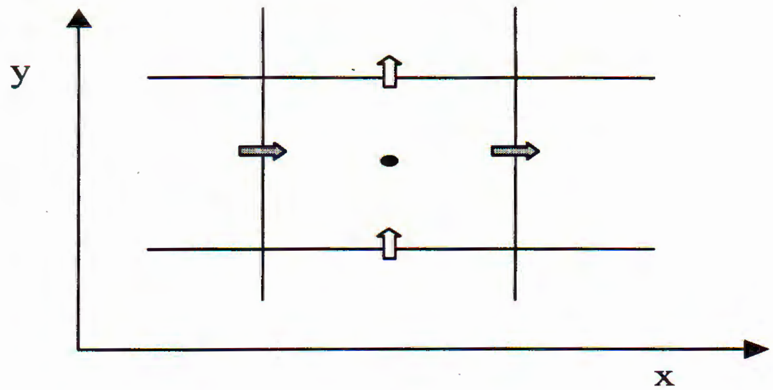
$$\begin{aligned}V_{i,h}^{n+1}&=\theta V_{i,j}^{n}+(1-\theta)\frac{1}{4}(C_{i,j+1}^{n}+V_{i,j-1}^{n}+V_{i+1,j}^{n}+V_{i-1,j}^{n})-\frac{\Delta t}{2\Delta x}\overline{U}_{ij}^{n}(V_{i+1,j}^{n}-V_{i-1,j}^{n})\\&-\frac{\Delta t}{2\Delta y}V_{ij}^{n}(V_{i,j+2}^{n}-V_{i,j-1}^{n})-g\frac{\Delta t}{2\Delta y}(\zeta_{i,j+1}^{n}-\zeta_{i,j-1}^{n})-\frac{\tau_{by}^{n}}{\rho h}\\&-\frac{1}{\rho h}\Big[\frac{\Delta t}{2\Delta y}(S_{yy_{1,j+1}}-S_{yy_{i,j-1}})+\frac{\Delta t}{2\Delta x}(S_{xy_{i+1,j}}-S_{xy_{i-1,j}})\Big]\end{aligned}$$

$$\begin{aligned}\zeta_{i,j}^{n+1}&=\zeta_{i,j}^{n}-\frac{\Delta t}{2\Delta x}\big[U_{i+1,j}(h_{i+1,j}+h_{i,j})-U_{i,j}(h_{i,j}+h_{i-1,j})\big]\\&-\frac{\Delta t}{2\Delta y}\big[V_{i,j+1}(h_{i,j+1}+h_{i,j})-V_{i,j}(h_{i,j}+h_{i,j-1})\big]\end{aligned}$$

where:

$$\begin{aligned}\overline{V}_{i,j}^{n}&=\frac{1}{4}(V_{i,j}^{n}+V_{i-1,j}^{n}+V_{i,j+1}^{n}+V_{i-1,j+1}^{n})\\\overline{U}_{i,j}^{n}&=\frac{1}{4}(U_{i,j}^{n}+U_{i,j-1}^{n}+U_{i,j+1}^{n}+U_{i-1,j+1}^{n})\\\tau_{bx}^{n}&=C_{f}U_{i,j}^{n}\sqrt{\left(U_{i,j}^{n}\right)^{2}+\left(V_{i,j}^{n}\right)^{2}}\\\tau_{by}^{n}&=C_{f}V_{i,j}^{n}\sqrt{\left(U_{i,j}^{n}\right)^{2}+\left(V_{i,j}^{n}\right)^{2}}\end{aligned}$$

(see also Fig. 1)



• is the point to calculate ζ
$\Uparrow$ is the point to be calculated U
$\Rightarrow$ is the point to be calculated V

Fig. 1. Difference scheme

4. Comparison between computed and observed values

The model is applied at the Phan Ri-Binh Thuan zone, in the latitude from 11°05′70N to 11°10′00N and in the longitude from 108°30′00E to 108°33′40E. The considered area is divided into the equal spatial steps of $dx = 50$ m (along the normal direction of shoreline) and $dy = 100$ m (along the shoreline).

The coefficients are chosen for the model, as follows [1]:

- Bottom friction coefficient: $C_f = 0.01$,
- The diffusion Lax parameter: $\theta = 0.95$,
- Water density: $\rho = 1.0\,\mathrm{kg/litre}$,
- Gravity acceleration: $g = 9.8\,\mathrm{m/s^2}$.

Let us consider the 5 cases of different deep-wave fields in South- West monsoon. The deep-wave parameters are received from the station in the depth of 20 m. The observed results of waves induced currents are received from the observed currents at the nearshore observation station. As known, the observed currents include tidal currents, wind driven currents and waves induced currents. Based on the synchronous observation of wind, tide, depth, tidal currents (at the deep wave observation station), waves induced currents component is separated from the observed currents. Some computed results in comparison with observed results which are correspondent with the 5 cases of different deep-wave fields are presented on the table 1. Figure 2 expresses the comparison between computed and observed results, the comparative results show a good agreement. Figure 3 expresses the distribution of waves induced currents field calculated by the two dimension model with parameters of deep-wave such as wave height $H_0 = 1.90$ m, wave period $T = 6.5$ s and approach wave direction SSE.

Table 1. Comparison between computed and observed wave induced currents at the nearshore station

cases	The input deep wave parameters: wave height (m)	wave period (s)	wave direction	observed current vel. [2](cm/s)	computed current vel. (cm/s)	error (%)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	1.2	5.9	SSE	14.9	17.2	15
2	1.1	5.5	SSE	12.5	13.5	8
3	1.0	5.4	SSE	10.4	12.0	15
4	1.4	5.5	SSE	18.5	16.1	13
5	1.7	5.6	SSE	24.6	22.6	8

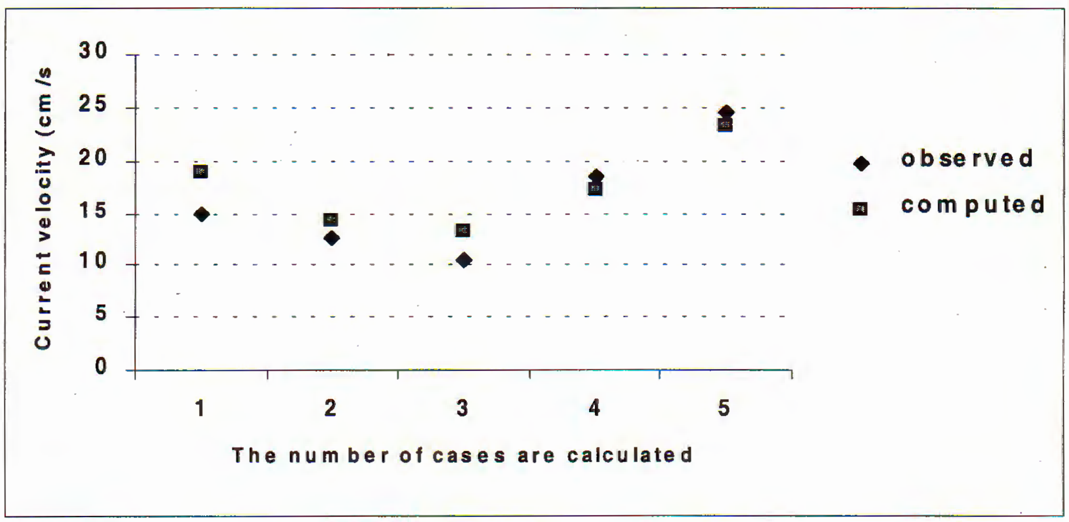


Fig. 2. Comparison between the computed and observed waves induced currents

5. The trial applying of model at the Ly-hoa zone of Quang Binh province

The Ly-hoa zone, one of many areas in Vietnam, is being eroded seriously. In the year 2001, CMESRC completed the project of shoreline protection for this zone. The research results bring out the project of building a stone jetty system for shoreline protection. Therefore, we calculated the distribution of waves induced currents with 2 cases as follows [3]: with and without stone jetty system. The RCPWAVE model was modified to calculate the difraction in the areas behind the construction.

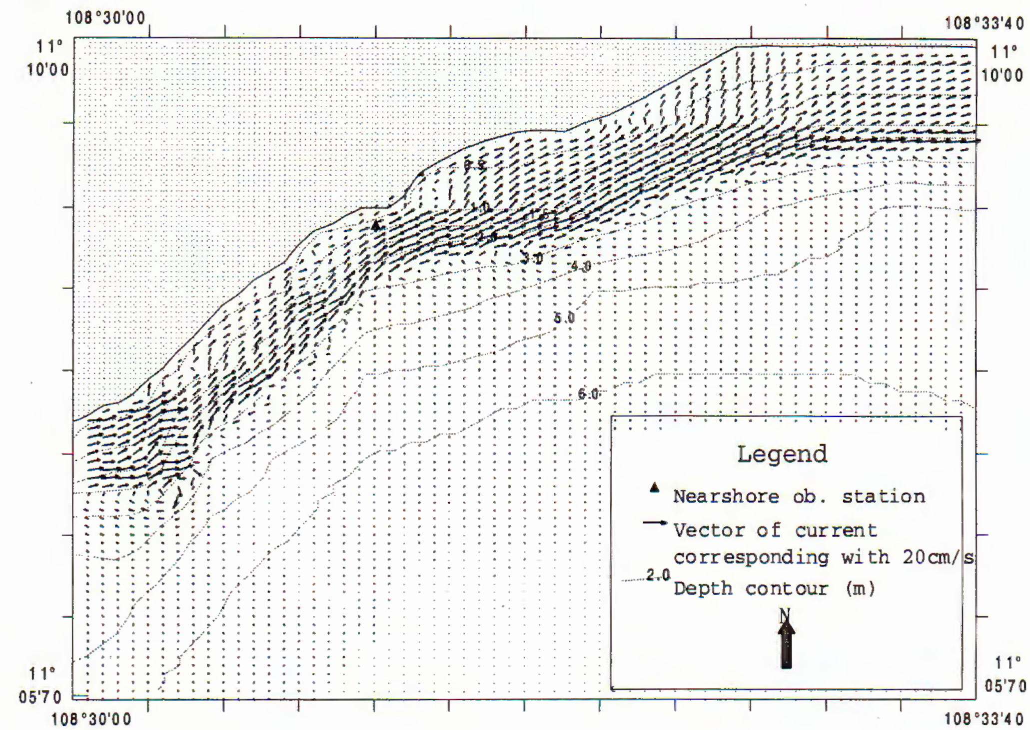


Fig. 3. Distribution of waves induced currents at the Phan Ri estuarine
Spatial step: $dx = 50\,\text{m}$, $dy = 100\,\text{m}$
(dx along the normal direction of shoreline and dy along the shoreline)

The parameters of deep-wave were applied to this model as follows: $H = 2.3\,\text{m}$, $T = 7.1\,\text{s}$, direction NE.

In the case of no stone jetty system (see fig. 4), the distribution of waves induced currents has shown some eroding areas. For instance, the Ly Hoa estuarine area, where there is rather strong possitive gradient of current, is being eroded. Thus, distribution of waves induced currents has contributed to the comprehension of eroding causes [3].

In the case of having stone jetty system (see fig. 5), the distribution of waves induced currents at jetty front and back is corresponded to some results received from other models [3].

6. Conclusions

- The testing results show a good agreement between the observed and computed values.

- Because the reflection is ignored in the model, it is advisable to apply the model

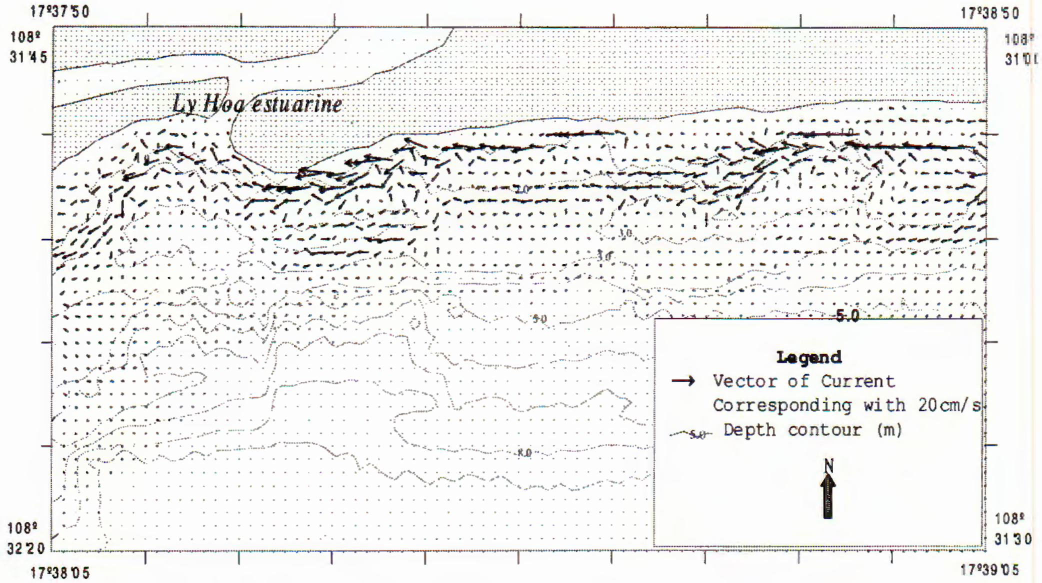


Fig. 4. Distribution of wave induced current at Ly Hoa estuarine of Quang Binh province
Spatial steps: $dx = 25\,\mathrm{m}$, $dy = 25\,\mathrm{m}$ (no stone jetty system)

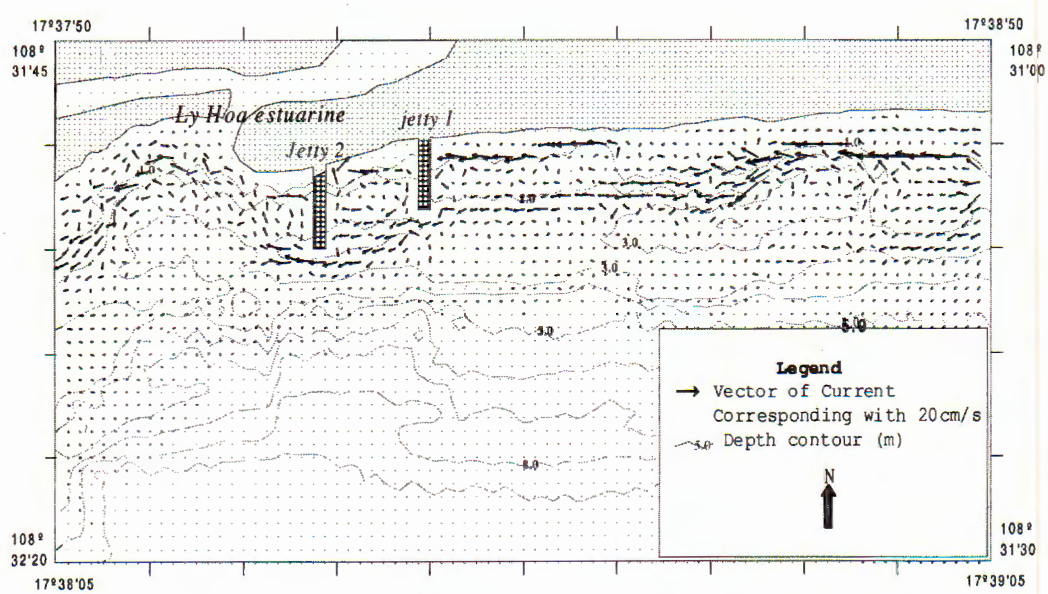


Fig. 5. Distribution of wave induced current at Ly Hoa estuarine of Quang Binh province
Spatial steps: $dx = 25\,\mathrm{m}$, $dy = 25\,\mathrm{m}$ (with the stone jetty systems)

with marine structures like groins or jetties which have the length perpendicular to the shoreline in order to reduce the reflecting waves

- The model needs testing more, then it can be applied to the calculation of waves induced currents in coastal zones of Vietnam.

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MÔ HÌNH HAI CHIỀU TÍNH DÒNG CHẢY DO SÓNG KHU VỰC GẦN BỜ

Bài báo trình bày kết quả của việc xây dựng mô hình số trị hai chiều để tính dòng chảy do sóng ở khu vực gần bờ. Đầu vào của mô hình là các tham số sóng khi sóng truyền vào vùng gần bờ. Mô hình RCPWAVE được sử dụng để tính phân bố các tham số sóng (mô hình này được xây dựng tại CMESRC). Vì mô hình RCPWAVE không tính đến hiệu ứng nhiễu xạ nên ở đây đã xây dựng thêm môđul để tính sóng nhiễu xạ phía sau công trình. Mô hình hai chiều tính dòng chảy do sóng được kiểm chứng bằng cách so sánh với kết quả đo đạc tại vùng biển Phan Rí - Bình thuận ứng với 5 trường hợp trường sóng ngoài khơi khác nhau, kết quả so sánh khá tốt. Mô hình được áp dụng tính toán thử cho vùng biển Lý Hòa - Quảng Bình trong trường hợp tự nhiên (khi chưa có các công trình bảo vệ bờ) và trong trường hợp sau khi xây dựng các công trình bảo vệ bờ biển. Bức tranh phân bố dòng chảy sóng trong trường hợp không có công trình đã góp phần lý giải được cơ chế xói lở nơi đây và trong trường hợp có công trình thì bức tranh dòng chảy sóng phù hợp với kết quả nhận được từ một số mô hình khác.